Technical Notes

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Multiple Slot Laminar Film Cooling

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BOUNDARY-LAYER injection as a means of reducing heat transfer and friction has been considered for a number of years. The injection can be accomplished by transpiration through a porous surface or by the use of slots. The present study considers injection through tangential slots into a laminar boundary layer. This study represents the initial phase of a theoretical program to investigate the feasibility of using film cooling in a Uranium plasma device. It was undertaken to aid in the development of the finite-difference techniques that are to be used in the later stages involving turbulent flow and coolants of different molecular weight than the fissioning core. The results are nevertheless quite interesting and not available heretofore.

For the results reported here, the injected gas is introduced through a slot parallel to the main flow and has the same molecular weight as the main flow. The slot lip thickness is assumed to be zero and the slot height is of the order of the external boundary-layer thickness. The gas is helium. The external main flow temperature is 8000°K while the wall temperature is taken to be 4000°K. The main flow velocity, u_∞, is 280 m/sec. The ratio of jet velocity to freestream velocity, u_j/u_∞, is a parameter in the analysis. The pressure is assumed to be everywhere constant. The location of the second slot is a function of the heat-transfer rate to the surface. The second slot is introduced when the heating to the surface exceeds a specified level. Based on these considerations, the governing conservation equations of mass, momentum, and energy are

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \tag{1}$$

$$\rho u\, \partial u/\partial x+\rho v\, \partial u/\partial y=(\partial/\partial y)(\mu\, \partial u/\partial y) \tag{2}$$

$$\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}=\frac{\mu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{k}{c_p}\frac{\partial T}{\partial y}\right) \tag{3}$$

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Index category: Boundary Layers and Convective Heat Transfer—Laminar.

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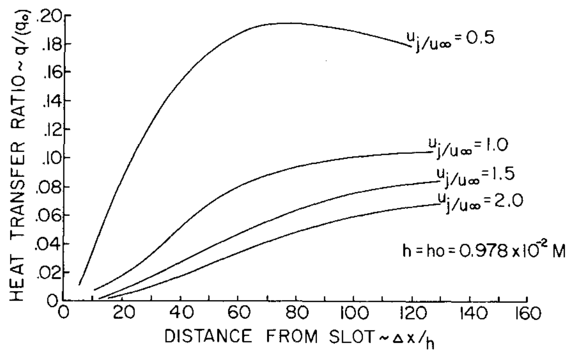

Fig. 1 Heat-transfer ratio vs distance from slot for various u_j/u_∞ ratios.

Using the Levy-Lees transformation

$$\eta(x,y)=\frac{U_\infty}{(2\xi)^{1/2}}\int_0^y \rho\, dy; \qquad \xi(x)=\rho_\infty\mu_\infty U_\infty x\,.$$

Eqs. (1–3) can be transformed into a coordinate system which would yield similar solutions in the absence of the slot.

A fixed wall temperature boundary condition was selected rather than the more conventional adiabatic condition because in developing a practical device one is concerned with determining the wall cooling required to maintain a specified temperature based upon material limitations.

The heat transfer to the wall is given by Eq. (4)

$$q=[u_\infty/(2\xi)^{1/2}Pr][(\partial\phi/\partial\eta)w](\rho\mu/\rho_\infty\mu_\infty) \tag{4}$$

where Pr = Prandtl number.

To establish a measure for the effectiveness of the film cooling technique, the injection heat-transfer rates have been calculated and results are presented in terms of the ratio of q/q_0 where q_0 is the zero injection heat-transfer rate at the first slot. The zero injection heat transfer as well as the initial boundary-layer profiles were calculated on the basis of a fully developed laminar boundary layer for which a similarity solution existed.

Of the many finite-difference procedures that can be employed to solve nonsimilar boundary-layer equations, the implicit procedure of Blottner[1] has been adopted in this study. The details describing the transformation from the partial differential equations to the finite-difference form are given in Ref. 2.

The slot height and starting point, x_0, are arbitrary; however, once selected they determine the relative size of the slot to the boundary layer in η coordinates. In the present study, the approach has been to pick a base slot height (η_0) in η equal to the boundary-layer thickness external to the slot and then

Table 1 Slot conditions

Single slot		Double slot: First slot		Double slot: Second slot	
u_j/u_∞	Slot height	u_j/u_∞	Slot height	u_j/u_∞	Slot height
0.5	h_0	1.0	h_0	0.25	$2.29h_0$
1.0	$0.5h_0-2.0h_0$	1.0	h_0	0.25	$1.145h_0, 2.29h_0$
1.5	h_0	1.0	h_0	1.0	$1.145h_0, 2.29h_0$

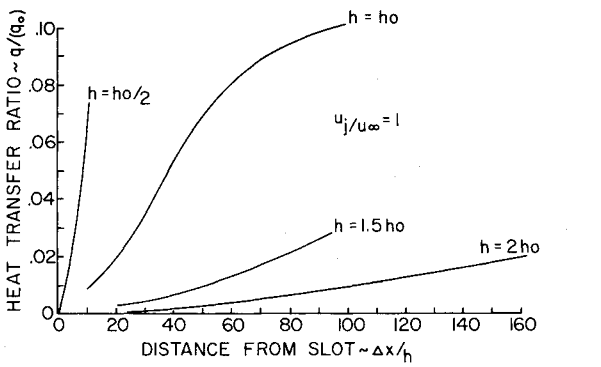

Fig. 2 Heat-transfer ratio vs distance from slot for various slot heights.

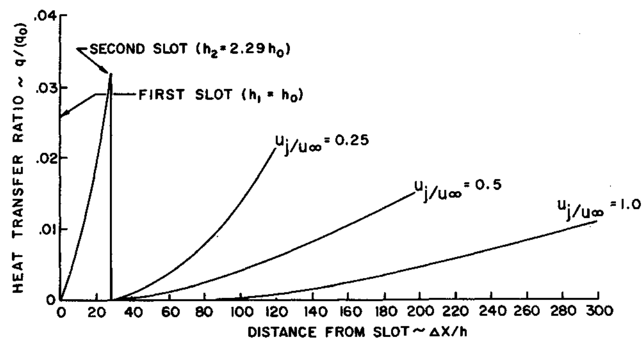

Fig. 3 Heat-transfer ratio vs distance from slot for double slot configuration.

select ξ_0 to obtain a dimensional slot height h_0 that was physically significant.

To examine the effect of slot height, jet velocity, and the second slot, a number of cases have been examined. These are shown in Table 1.

Figure 1 shows the effect of jet velocity on heat transfer to the surface. The peak in Fig. 1 for $u_j/u_\infty = 0.5$ is due to the $\xi^{-0.5}$ variation of the heat transfer [i.e., Eq. (4)]. The effect of slot height is shown in Fig. 2. Here, it can be seen that slot height is much more important in reducing heat transfer than jet velocity. In Fig. 2, the distance to $q/q_0 = 0.02$ for $h = 2h_0$ is $162h_0$. For the same coolant flow in Fig. 1, $q/q_0 = 0.02$ is reached at a distance of $44h_0$ for $u_j/u_\infty = 2.0$.

Figure 3 presents the results for a double slot configuration. The location of the second slot is dictated by design limitations on the heat transfer to the wall. For an 8000°K main flow temperature, the black body radiation which is not affected by film cooling is 2.25 kw/cm^2 and the zero injection convection at the slot is approximately 18 kw/cm^2. Assuming a maximum allowable heating of 3 kw/cm^2, it is reasonable to limit the convection with film cooling to 25% of the radiation heating.

This condition requires that the second slot be introduced 20 slot heights, downstream of the first slot. The jet velocity ratio at the second slot was varied from 0.25 to 1.0. The second slot was the same height as the first in η coordinates or $2.29h_0$ in physical dimensions. From Fig. 3, it can be seen that the second slot is more effective than the first. For a $u_j/u_\infty = 1.0$ at the second slot, the distance protected is increased by a factor of ten ($300h_0$ vs $30h_0$).

In film cooling, the most significant parameter is the coolant mass flow required. For the cases considered in this study, the coolant mass flow is proportional to

$$[(u_j/u_\infty)_1 h_1 + (u_j/u_\infty)_2 h_2]$$

Table 2 shows comparison of the protection ($q/q_0 < 0.032$) provided by three different film cooling configurations.

This study has provided results for film cooling by means of tangential injection into a laminar boundary layer. The calculations have shown that for the same coolant mass flow increasing slot thickness is a more effective means of reducing heat transfer than increasing jet velocity. The introduction of a second slot can provide a significant increase in protection when compared to the single slot cases.

The next phase of this study is the examination of the turbulent case. Turbulent mixing tends to be more rapid than laminar mixing, thereby reducing the cooling effectiveness with distance. However, the turbulent heat-transfer rates with or without injection decrease as $\xi^{-0.8}$ compared to $\xi^{-0.5}$ for laminar flow; hence the two effects tend to counteract each other so that qualitatively the results obtained herein might be indicative of what happens for turbulent flow.

Table 2 Comparison of protection methods

Configuration	Distance protected	Coolant mass flow parameter
Single slot		
($h = h_0$, $u_j/u_\infty = 2.0$)	$58h_0$	2
($h - 1.5h_0$, $u_j/u_\infty = 1.0$)	$100h_0$	1.5
Double slot		
$h_1 = h_0$, $u_j/u_\infty = 1.0$ $h_2 = 2.29h_0$, $u_j/u_\infty = 0.5$	$140h_0$	1.57

References

[1] Blottner, F. G., "Non-Equilibrium Laminar Boundary Layer Flow of a Binary Gas," GE TIS R63SD17, June 1963, General Electric Co., Philadelphia, Pa.

[2] Sherman, A., Yeh, H., McAssey, E., Jr., and Reshotko, E., "Summary Report MHD Boundary Layers with Non-Equilibrium Ionization and Finite Rates," June 1970, Computer and Applied Sciences Inc., Norristown, Pa.

Application of Nonlinear Estimators to Hereditary Systems

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BECAUSE of the increasing importance of dynamic processes with time delays, the study of the optimal control and estimation of hereditary systems has been intensified during the past decade. Various authors have shown the effect of the dead time to be important in such diverse problems as the modeling

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